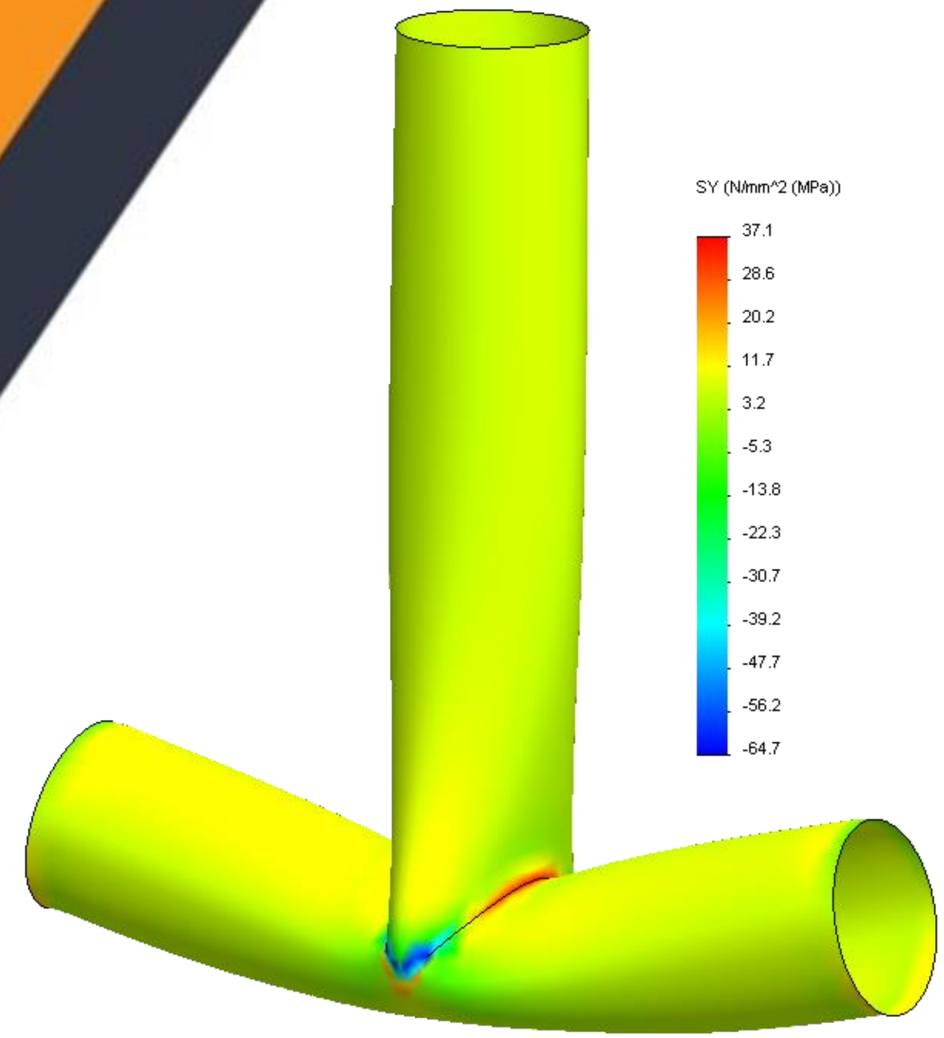


Strength of Materials II

Chapter 1

Combined Stresses

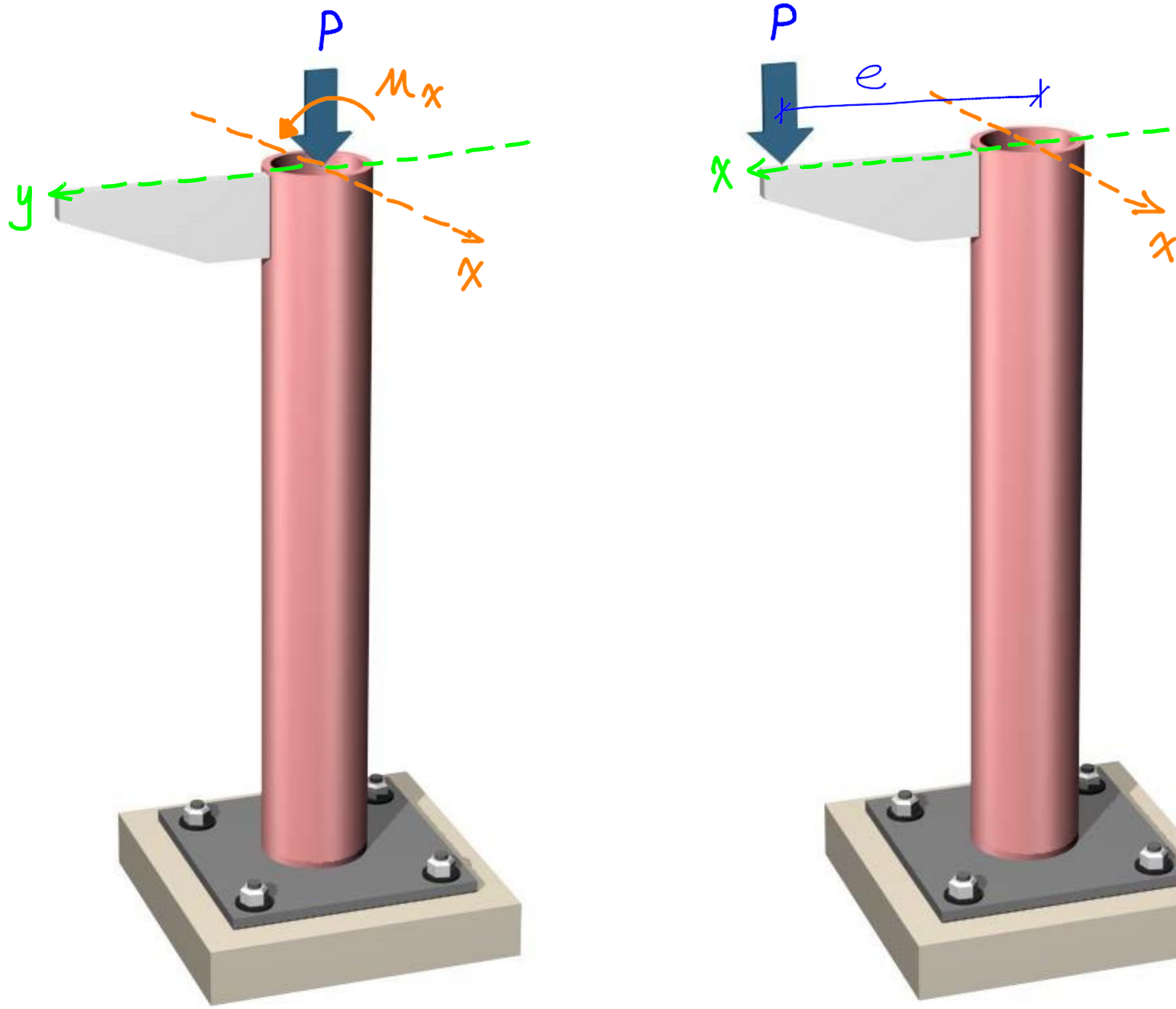
طريقك إلى إنجاح
YOUR WAY TO SUCCESS

Combined loading (bending moment + axial force)

Eccentric loading causes bending moment.

$M = P\,e$

M: Bending moment
P: Eccentric axial load
e: distance of load from centroid of section

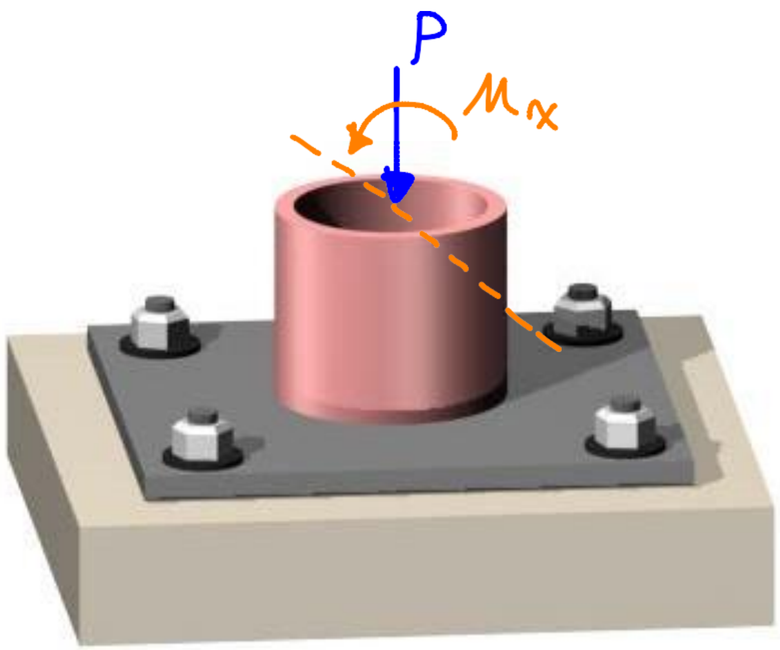


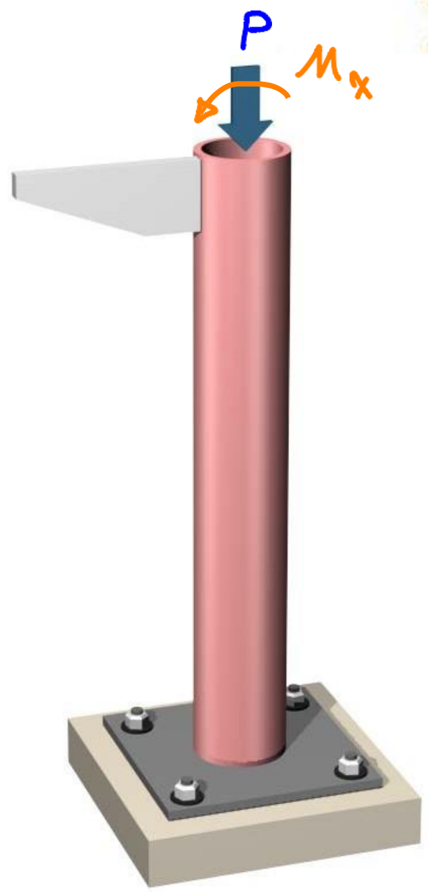
Combined loading (bending moment + axial force)

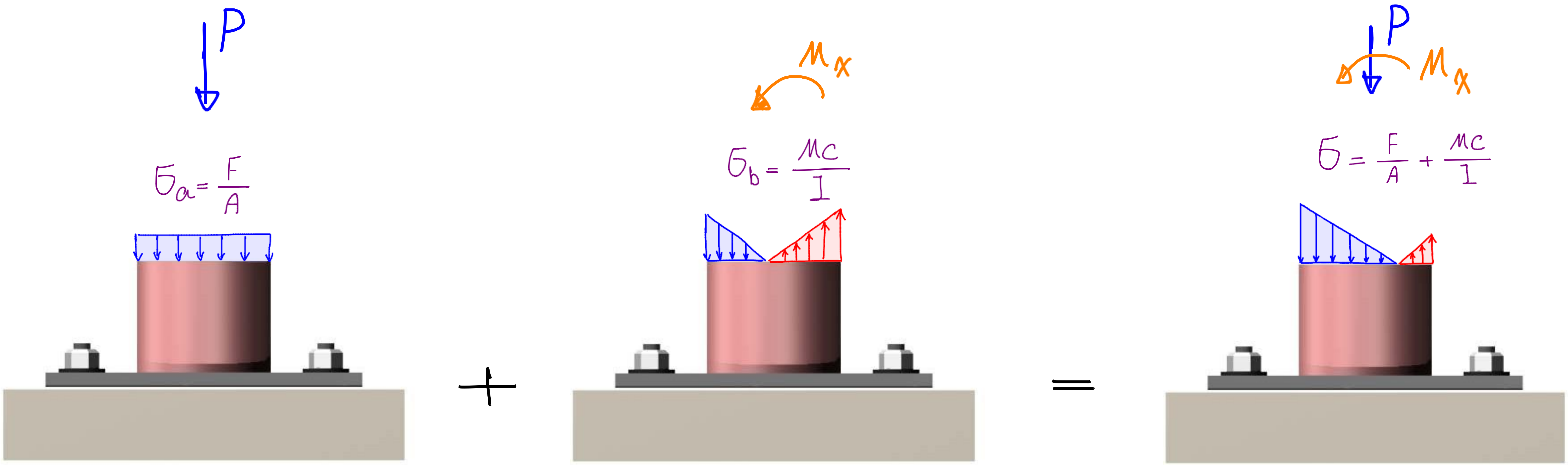
$$\sigma_a = \frac{F}{A}$$ Normal stress caused by axial force

$$\sigma_b = \pm\frac{Mc}{I}$$ Normal stress caused by bending moment

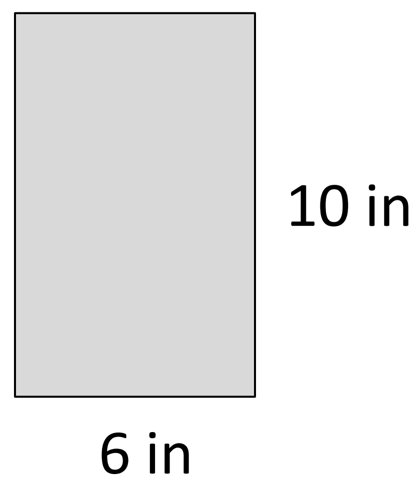
$$\sigma = \frac{F}{A} \pm \frac{Mc}{I}$$ Total normal stress

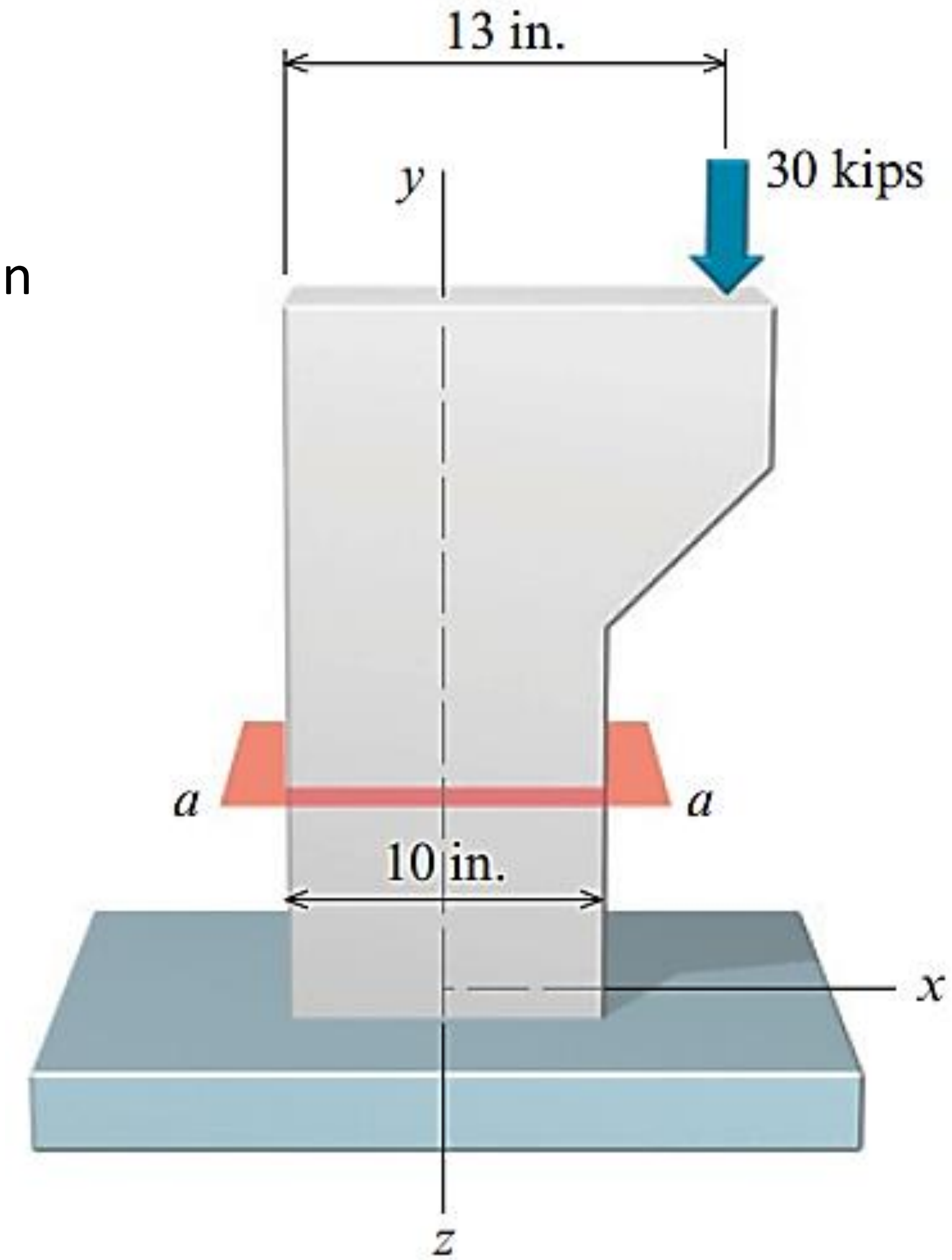






<u>Example 1:</u> A structural member with a rectangular cross section 10 in. wide by 6 in. deep supports a 30 kip concentrated load as shown in the figure. Determine the distribution of normal stresses on section a–a of the member.





Section Properties

The centroid location is known from symmetry.

The area of the cross section is A = (10 in.)(6 in.) = 60 in.^{2}.

$$I_z = \frac{(6 \text{ in.})(10 \text{ in.})^3}{12} = 500 \text{ in.}^4$$

Axial Stress

On section a–a, the internal force F = 30 kips

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{30 \text{ kips}}{60 \text{ in.}^2} = 0.5 \text{ ksi (C)}$$

Bending Stress

internal bending moment M = F × e = (30 kips)(8 in.) = 240 kip ·in

The magnitude of the maximum bending stress on section a–a can be determined from the flexure formula:

$$\sigma_{\text{bend}} = \frac{Mc}{I_z} = \frac{(240 \text{ kip}\cdot\text{in.})(5 \text{ in.})}{500 \text{ in.}^4} = 2.4 \text{ ksi}$$

Example 1: A structural member with a rectangular cross section 10 in. wide by 6 in. deep supports a 30 kip concentrated load as shown in the figure. Determine the distribution of normal stresses on section a–a of the member.

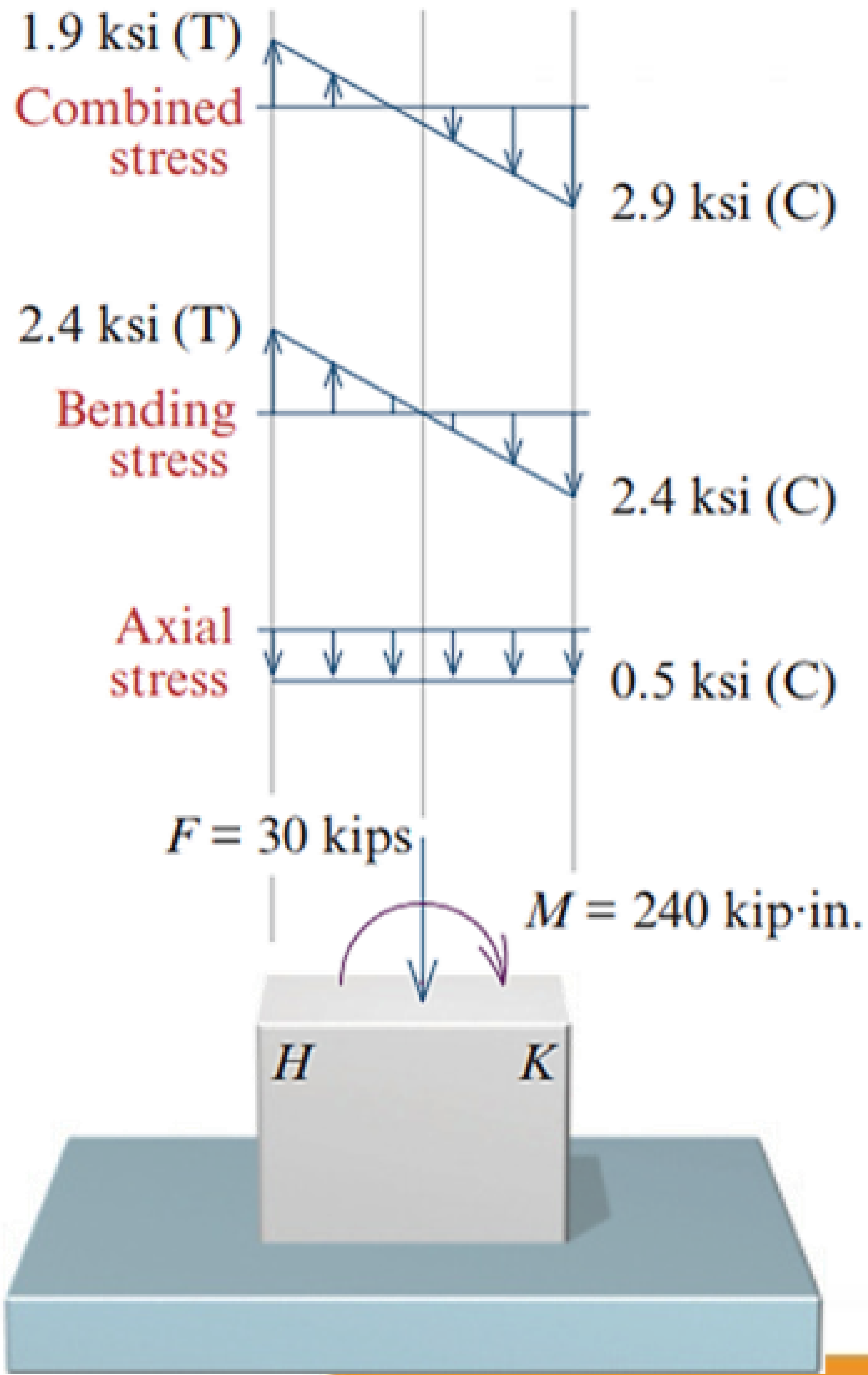
Combined Normal Stresses

The combined normal stress on side H of the structural member is:-

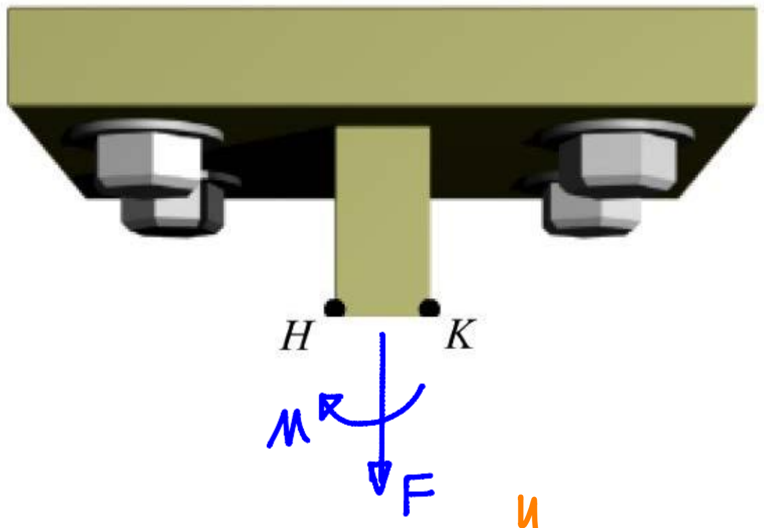
$$\sigma_H = \sigma_{\text{axial}} + \sigma_{\text{bend}} = -0.5 \text{ ksi} + 2.4 \text{ ksi} = +1.9 \text{ ksi} = 1.9 \text{ ksi (T)}$$

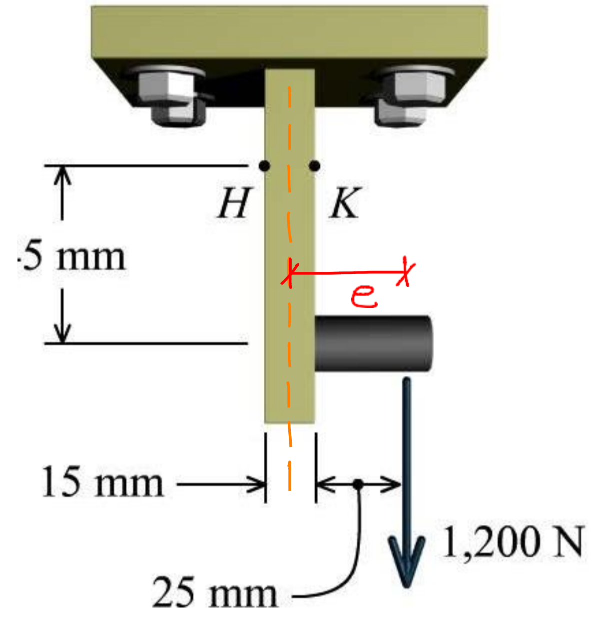
and the combined normal stress on side K is:-

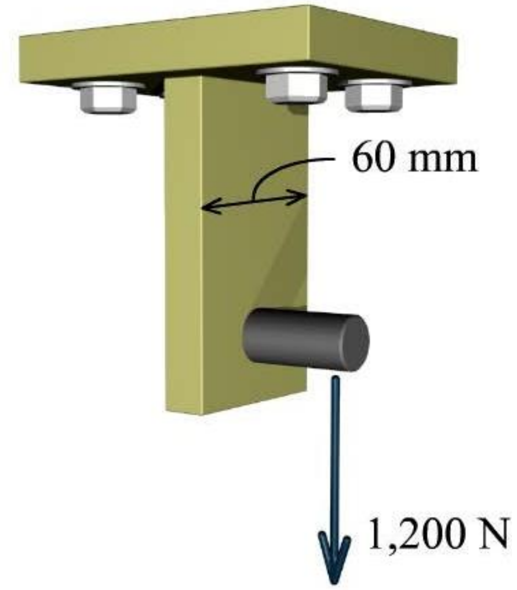
$$\sigma_K = \sigma_{\text{axial}} + \sigma_{\text{bend}} = -0.5 \text{ ksi} - 2.4 \text{ ksi} = -2.9 \text{ ksi} = 2.9 \text{ ksi (C)}$$



Example 2: A pin support consists of a vertical plate 60 mm wide by 15 mm thick. The pin supports a load of 1,200 N. Determine the normal stresses acting at points H and K







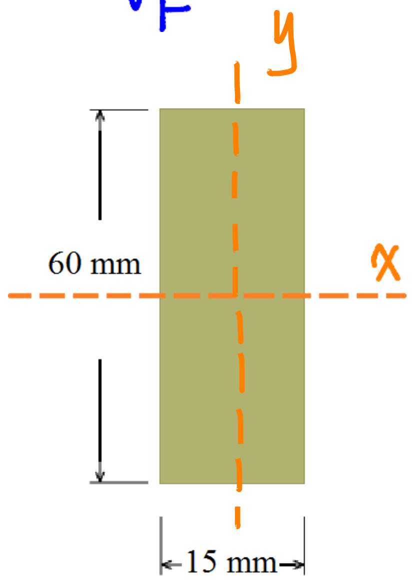
Step (1) Force and moment at base

$F = 1200\ N$

$M = F \times e$

$= 1200\ N \times (25 + \frac{15}{2})$

$= 39{,}000\ N.mm$



Step (2) Section properties

$A = bh = 900\ mm^2$

$I_y = \frac{bh^3}{12} = 16{,}875\ mm^4$

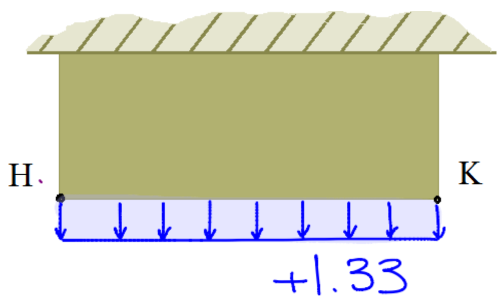
$b = 60\ mm,\ h = 15\ mm$

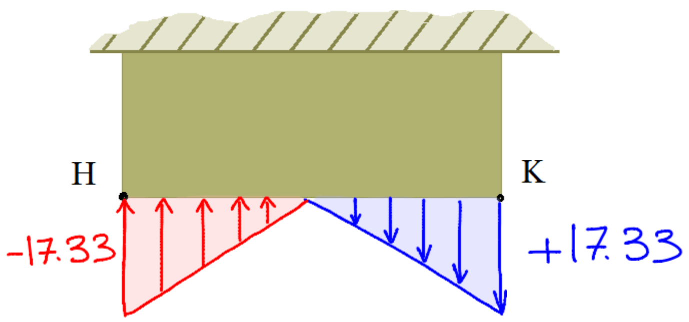
Step ③

$\sigma_a = \frac{F}{A} = 1.33\ mpa$

$\sigma_b = \frac{MC}{I} = \frac{39{,}000 \times 15/2}{16{,}875\ mm^4}$

$= \pm 17.33$

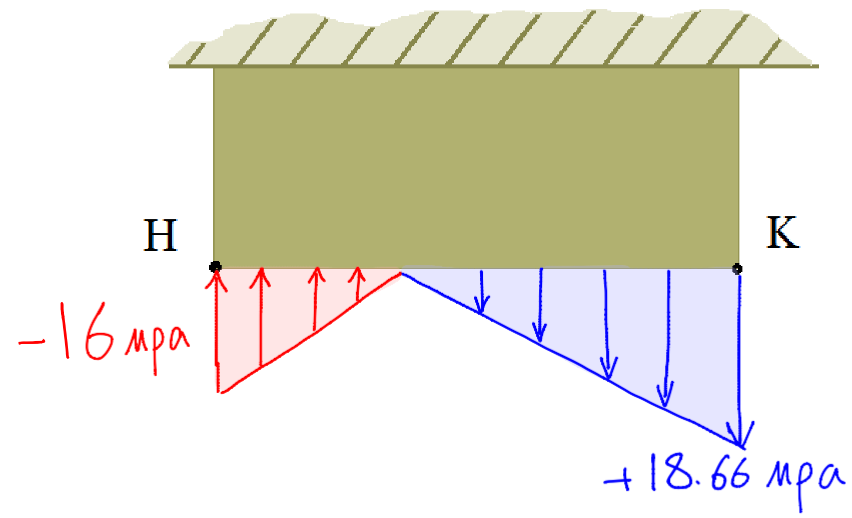




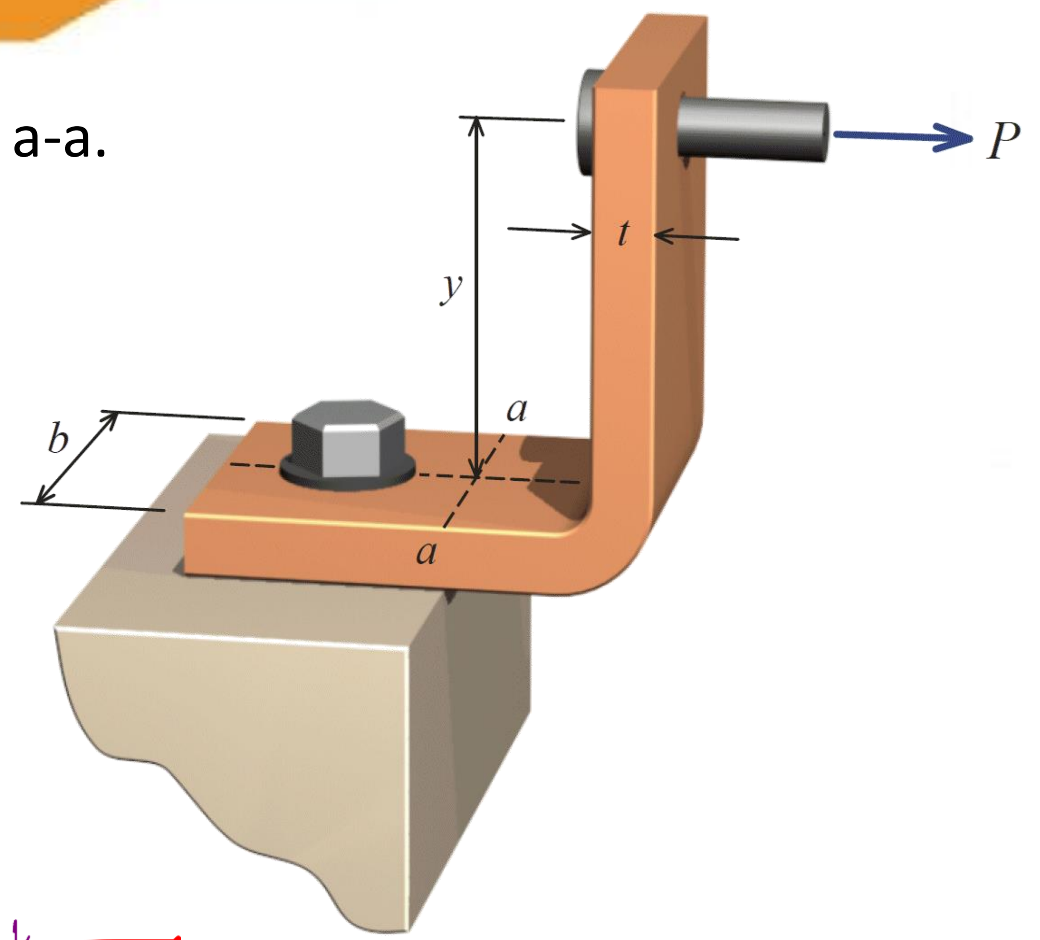
Step (5) Total stress at K and H

$\sigma_K = \sigma_a + \sigma_b = 18.66\ mpa$

$\sigma_H = \sigma_a - \sigma_b = -16.00\ mpa$



Example 3: Determine the maximum normal stress at section a-a.
P = 450 lb,   t= 1/2 in.,   b = 3 in.,   y = 1 ft



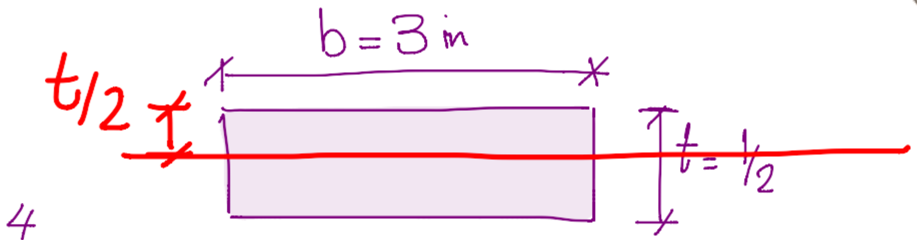
Step ① Moment and force

$$F = P = 450 \text{ lb}$$

$$M = Pe = 450 \times 12.25 = 5513 \text{ lb-in}$$

$$e = y + \frac{t}{2} = 12.25 \text{ in}$$

Step ② Section properties



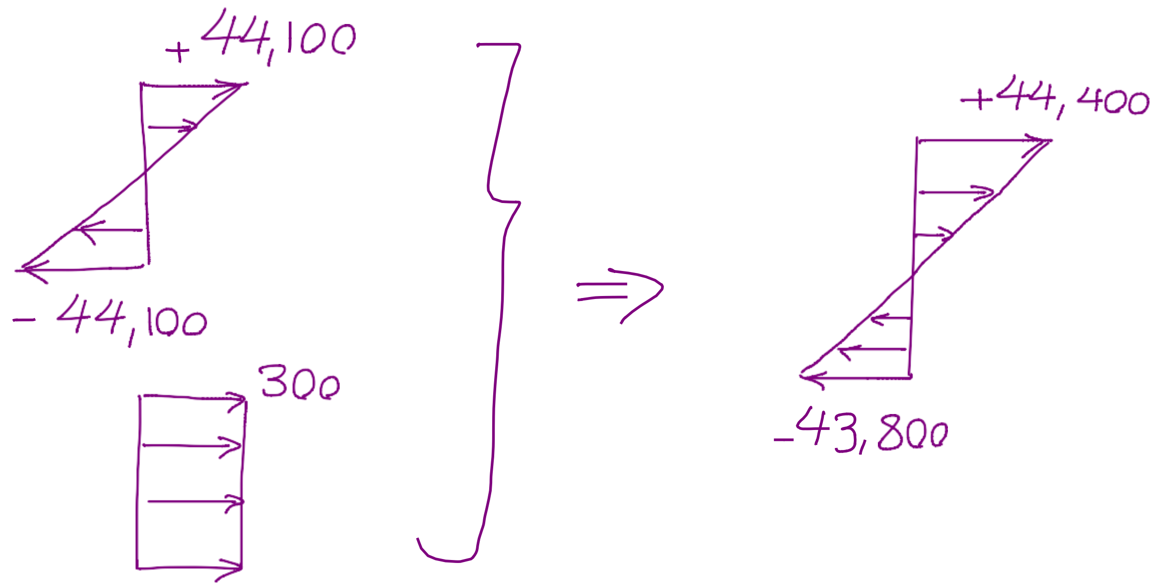
$$I = \frac{3 \times (1/2)^3}{12} = 0.03125 \text{ in}^4$$

Step ③ Stress

$$\sigma_b = \frac{Mc}{I} = \frac{5513 \times 1/4}{0.03125} = \boxed{44,100 \text{ psi}}$$

$$\sigma_a = \frac{F}{A} = \frac{450}{3 \times 1/2} = \boxed{300 \text{ psi}}$$

$$\sigma = \sigma_a + \sigma_b = \boxed{44,400 \text{ psi}}$$

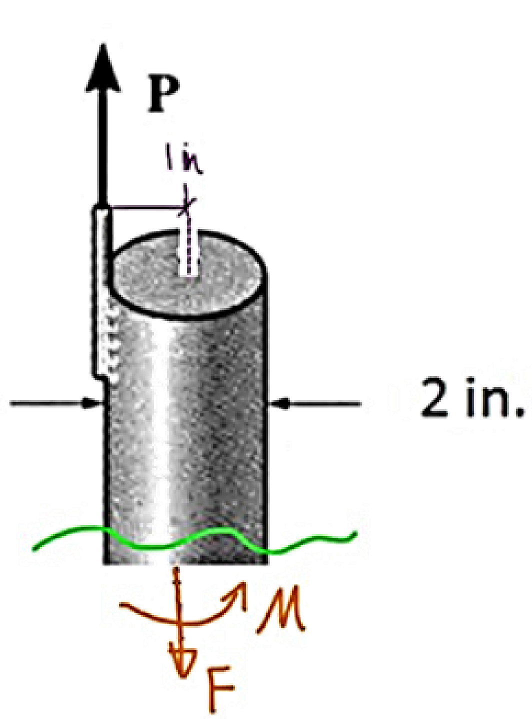


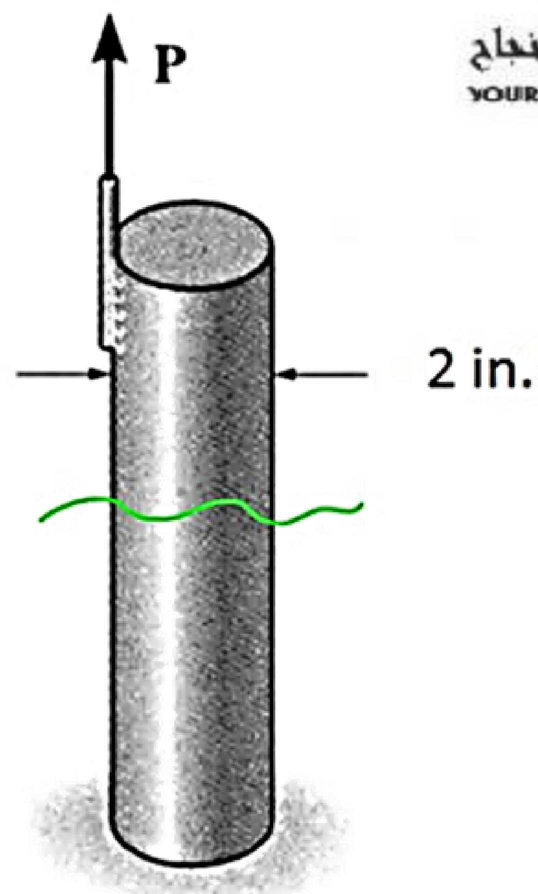
Example 4:

Determine the maximum load *P* that can be applied so that the normal stress in the rod does not exceed 12 ksi.

$$\Sigma F_y = 0 \Rightarrow F = P$$

$$\Sigma M_o = 0 \Rightarrow P \times 1 = M$$





Section properties

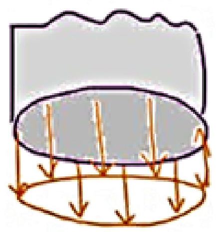
$$A = \frac{\pi d^2}{4} = 3.14 \text{ in}^2$$

$$I = \frac{\pi d^4}{64} = 0.785 \text{ in}^4$$

$$C = \frac{d}{2} = 1 \text{ in}$$

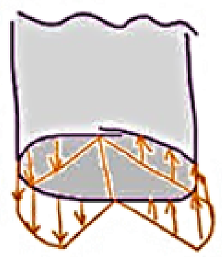
Axial stress

$$\sigma = \frac{F}{A} = \frac{P}{3.14}$$



Stress casued by moment

$$\sigma = \frac{MC}{I} = \pm \frac{P \times 1}{0.785}$$

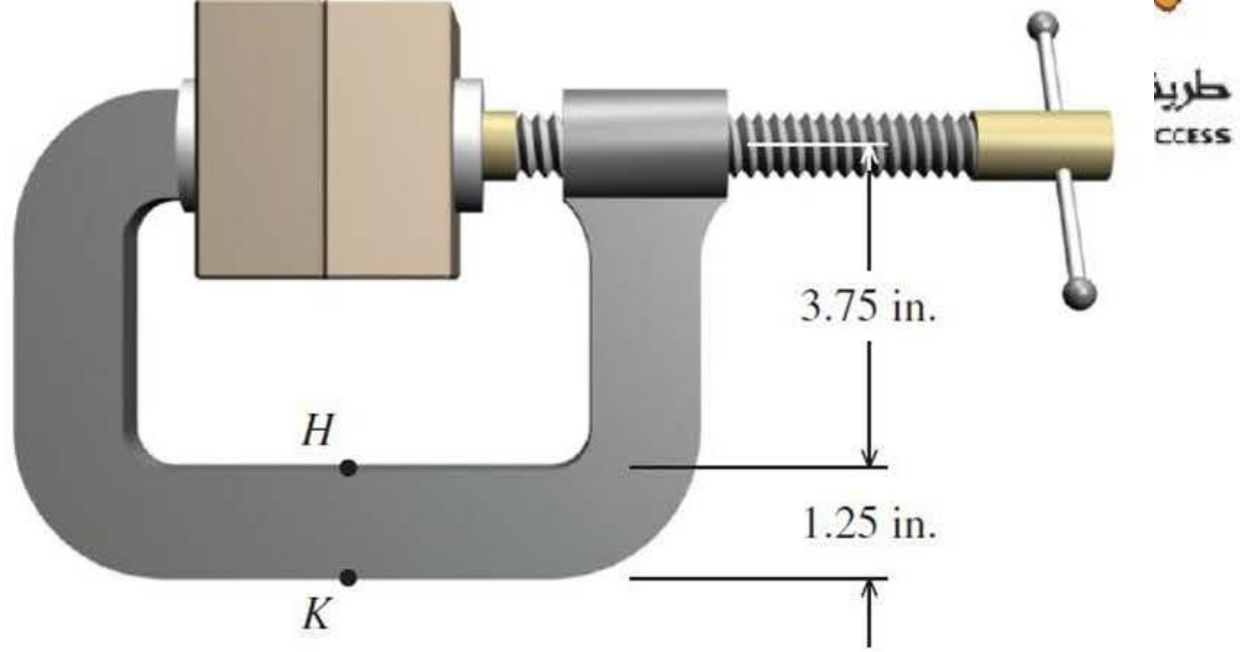


$$\sigma = \frac{F}{A} + \frac{MC}{I} = \frac{P}{3.14} + \frac{P}{0.785} = 1.592P \leq 12 \text{ ksi} = 12{,}000 \text{ psi}$$

$$P \leq \frac{12{,}000}{1.592} = \boxed{7540 \text{ lb}}$$

Example 5:

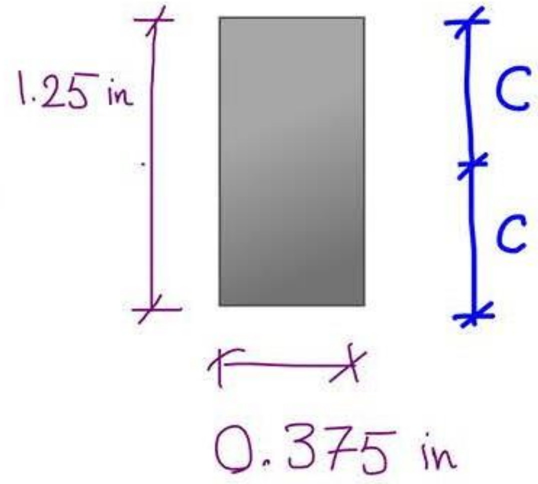
The screw of a clamp exerts a compressive force of 350 lb on the wood blocks. Determine the normal stresses produced at points H and K. The clamp cross-sectional dimensions at the section of interest are 1.25 in. by 0.375 in. thick.



$$\sum F_x = 0 \Rightarrow F = 350\ lb$$

$$\sum M_A = 0 \Rightarrow -M + 350\ lb \times \left(3.75 + \frac{1.25}{2}\right) in = 0$$

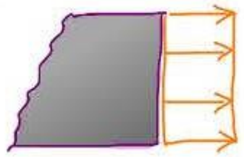
$$M = +1531\ lb.in$$

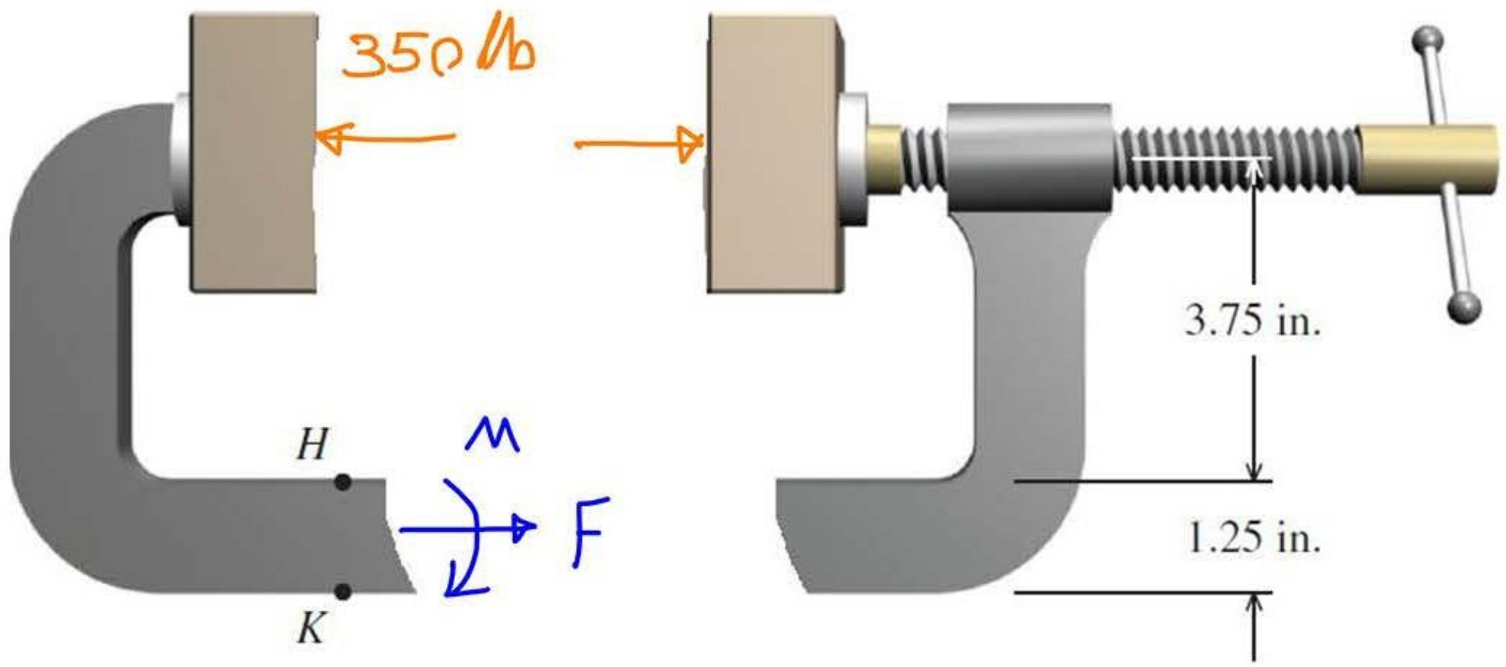


Step (1) Stress caused by axial force

$$\sigma_1 = \frac{F}{A} = \frac{350\ lb}{469\ in^2} = \boxed{+746\ psi}$$ Tension

$$A = 0.375 \times 1.25 = 0.469\ in^2$$





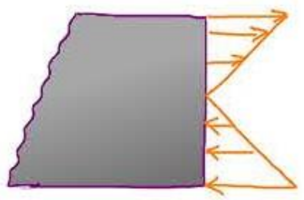
Step (2) Stress caused by bending moment

$$\sigma_2 = \frac{Mc}{I} = \frac{1531\ lb.in \times 0.625\ in}{0.061\ in^4} = \boxed{\pm 15686\ psi}$$

Tension and compression

$$I = \frac{bh^3}{12} = \frac{0.375 \times 1.25^3}{12} = 0.0610\ in^4$$

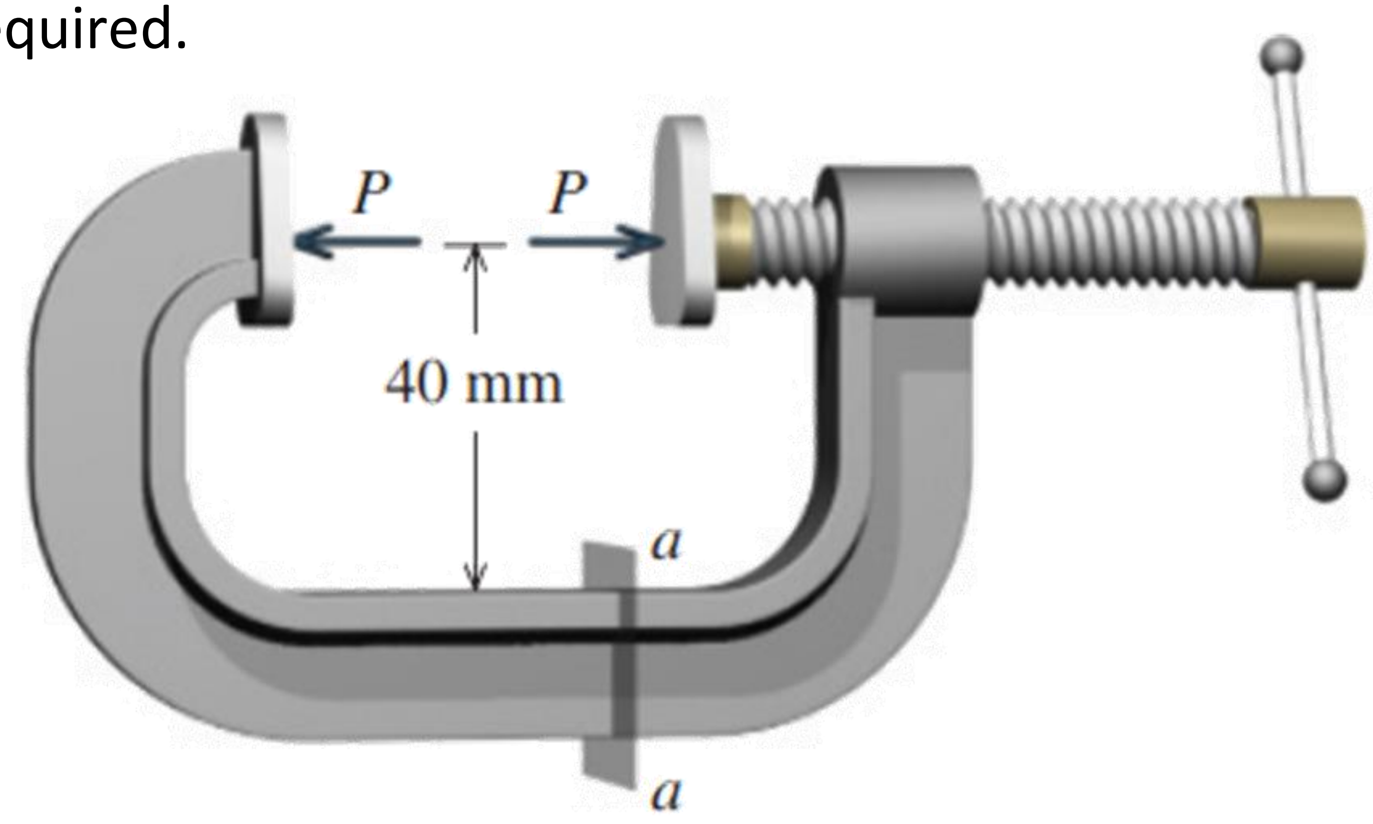
$$y = \frac{1.25}{2} = 0.625\ in$$

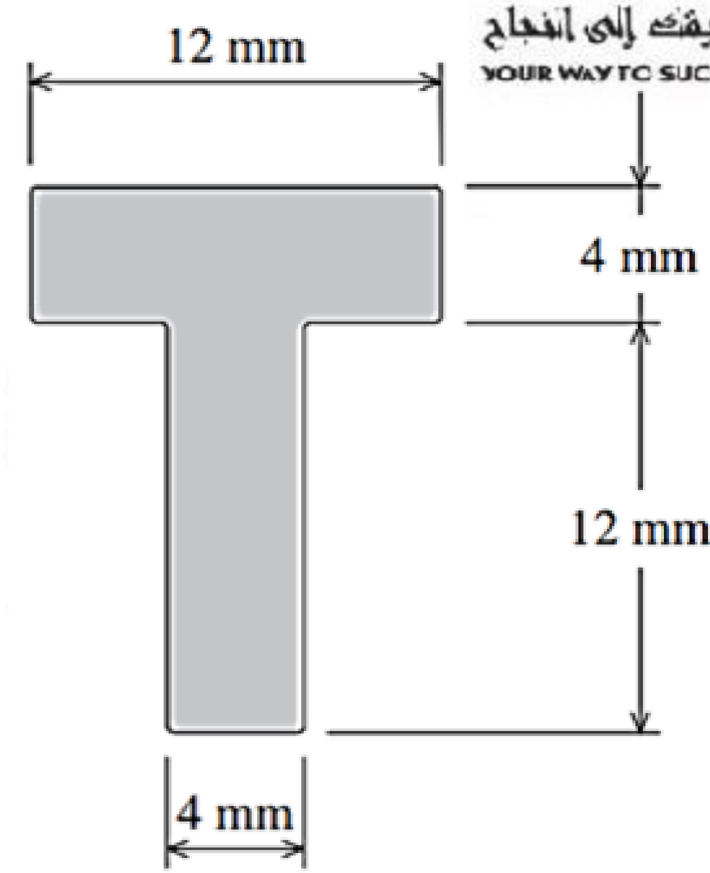


Normal stress at k: $\sigma_k = 746 - 15686 = \boxed{-14,940\ psi}$

Normal stress at H: $\sigma_H = 746 + 15686 = \boxed{+16,432\ psi}$

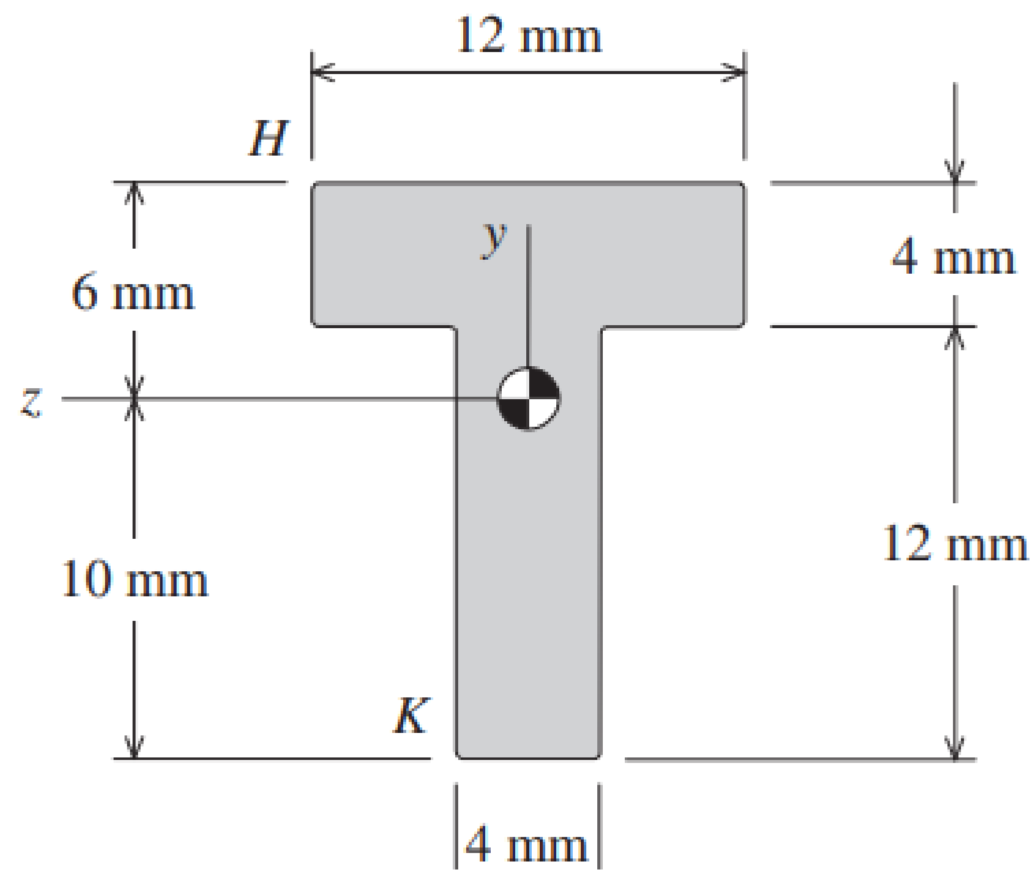
<u>Example 6:</u> The C-clamp shown is made of an alloy that has a yield strength of 324 MPa in both tension and compression. Determine the allowable clamping force that the clamp can exert if a factor of safety of 3.0 is required.





Section Properties

The centroid for the tee-shaped cross section is located as shown in the sketch on the left. The cross-sectional area is $A = 96\ mm^2$, and the moment of inertia about the z axis is calculated as $I_z = 2{,}176\ mm^4$.



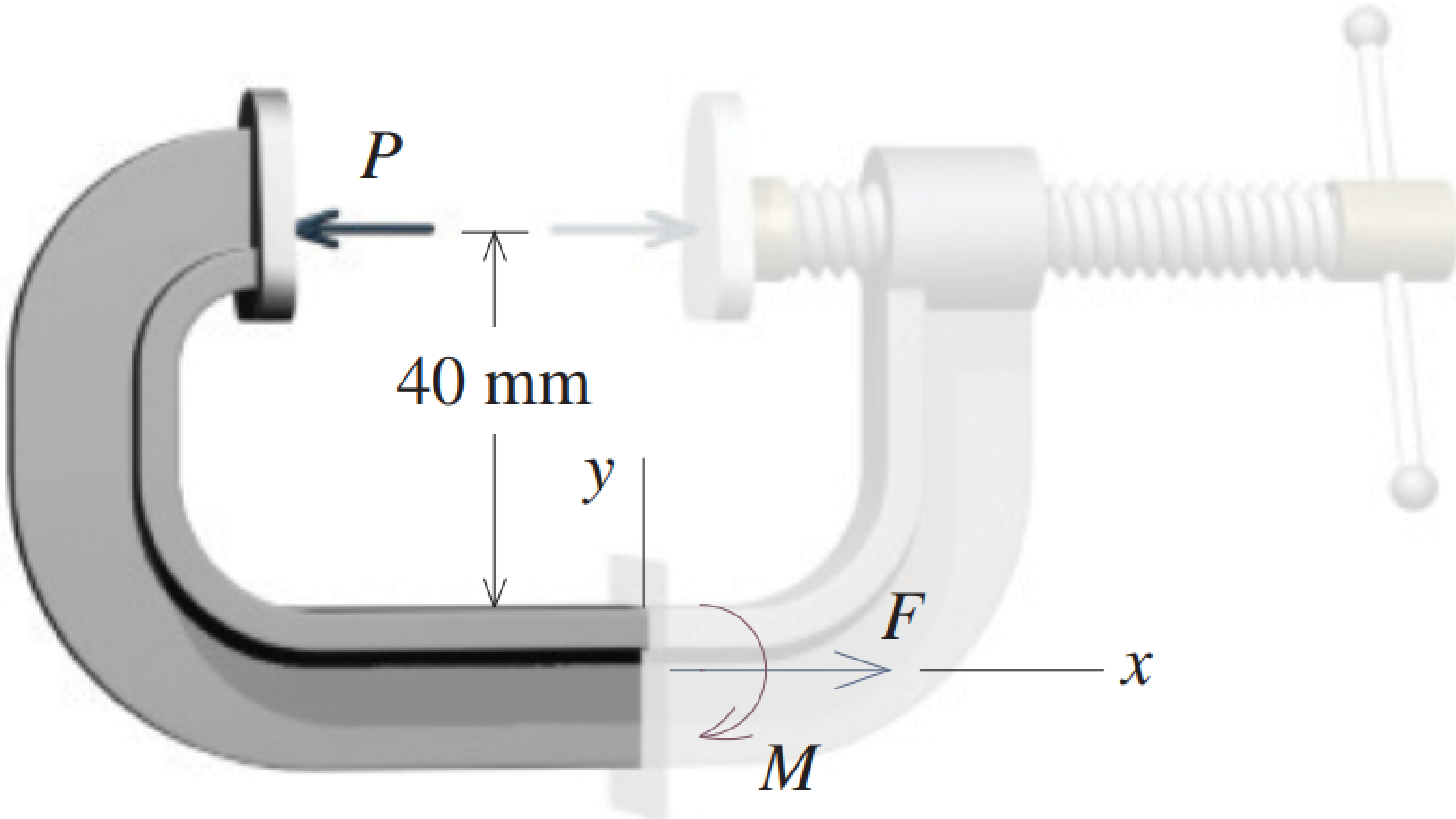
Allowable Normal Stress

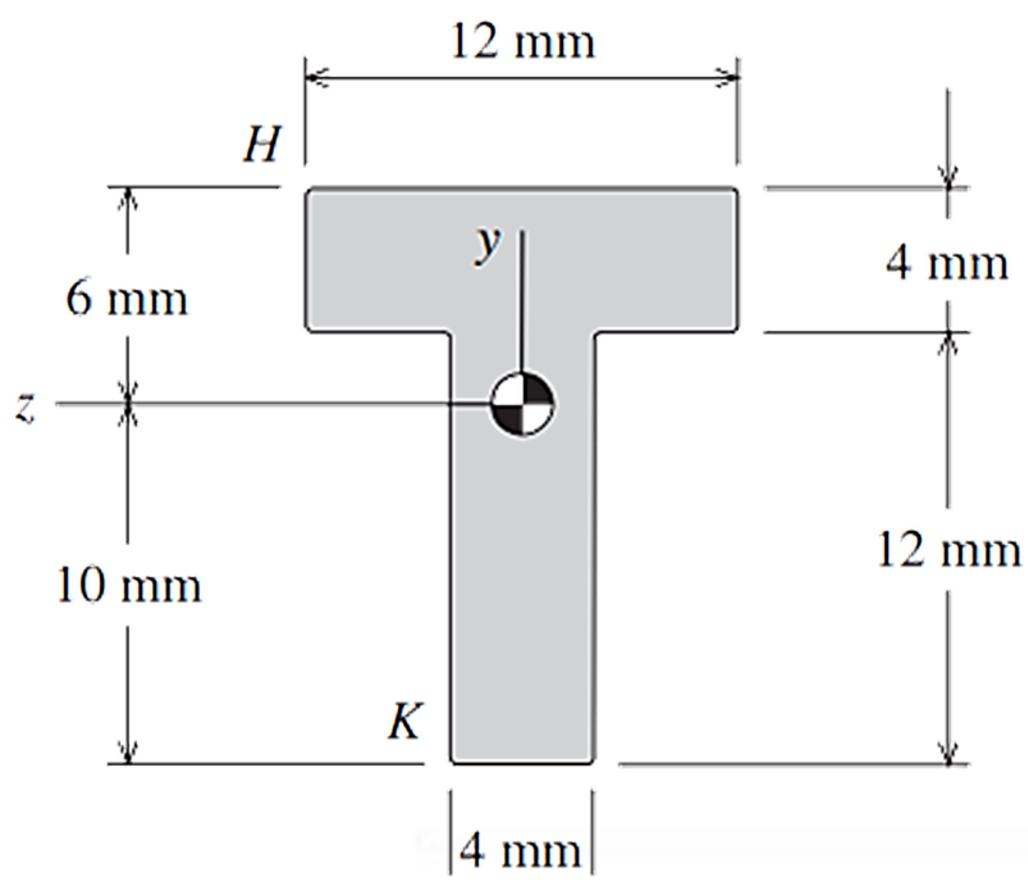
The alloy used for the clamp has a yield strength of 324 MPa. Since a factor of safety of 3.0 is required, the allowable normal stress for this material is 108 MPa.

<u>Example 6:</u> The C-clamp shown is made of an alloy that has a yield strength of 324 MPa in both tension and compression. Determine the allowable clamping force that the clamp can exert if a factor of safety of 3.0 is required.

Internal Force and Moment

A free-body diagram cut through the clamp at section a–a is shown.
The internal axial force F is equal to the clamping force P. The internal bending moment M is equal to the clamping force P times the eccentricity e = 40 mm + 6 mm = 46 mm between the centroid of section a–a and the line of action of P.





<u>Example 6:</u> The C-clamp shown is made of an alloy that has a yield strength of 324 MPa in both tension and compression. Determine the allowable clamping force that the clamp can exert if a factor of safety of 3.0 is required.

Axial Stress

On section a–a, the internal force F (which is equal to the clamping force P) produces a normal stress

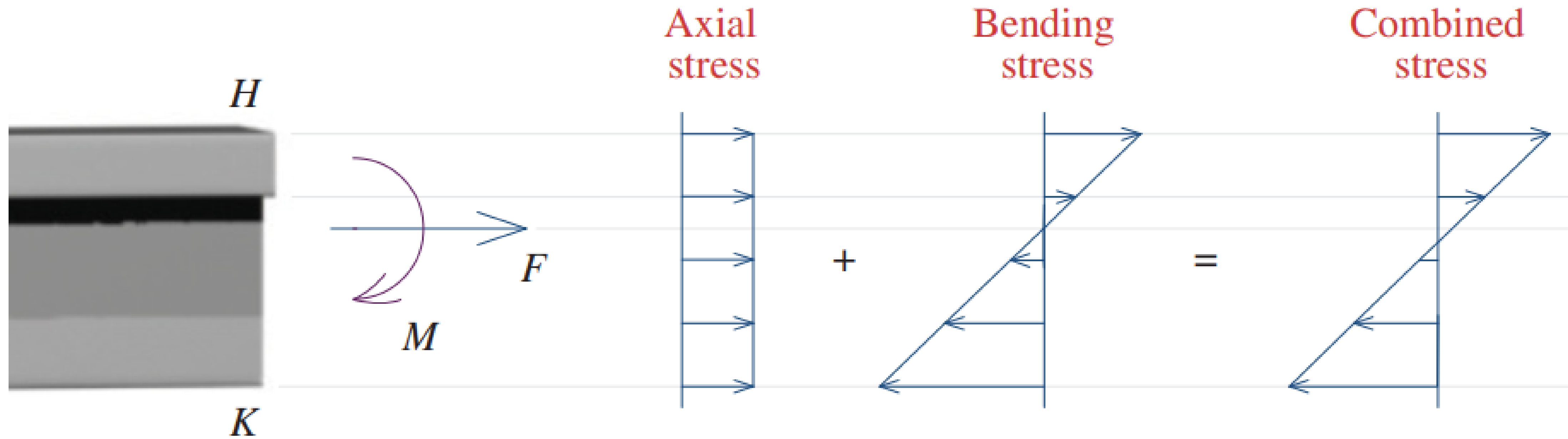
$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{P}{A} = \frac{P}{96\ \text{mm}^2}$$

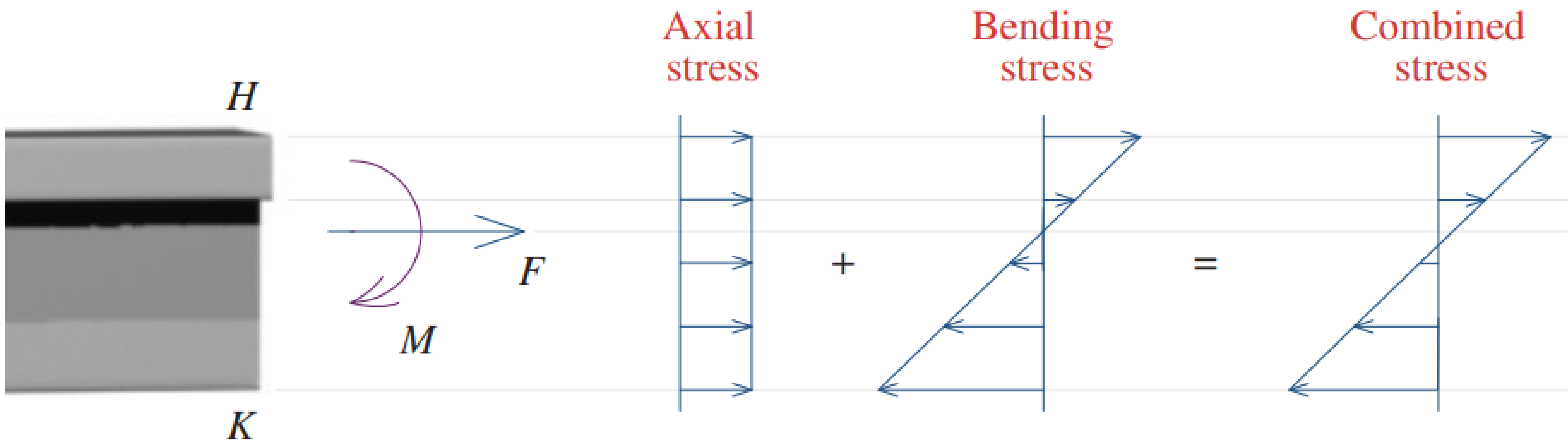
Bending Stress

Since the tee shape is not symmetrical about its z axis, the bending stress on section a–a at the top of the flange (point H) will be different from the bending stress at the bottom of the stem (point K).

$$\sigma_{\text{bend},H} = \frac{My}{I_z} = \frac{P(46\ \text{mm})(6\ \text{mm})}{2{,}176\ \text{mm}^4} = \frac{P}{7.88406\ \text{mm}^2}$$

$$\sigma_{\text{bend},K} = \frac{My}{I_z} = \frac{P(46\ \text{mm})(10\ \text{mm})}{2{,}176\ \text{mm}^4} = \frac{P}{4.73043\ \text{mm}^2}$$





Combined Stress at H

$$\sigma_{\text{comb},H} = \frac{P}{96\text{ mm}^2} + \frac{P}{7.88406\text{ mm}^2} = P\left[\frac{1}{96\text{ mm}^2} + \frac{1}{7.88406\text{ mm}^2}\right] = \frac{P}{7.28572\text{ mm}^2}$$

$$\frac{P}{7.28572\text{ mm}^2} \leq 108\text{ MPa} = 108\text{ N/mm}^2 \qquad \therefore P \leq 787\text{ N}$$

Combined Stress at K

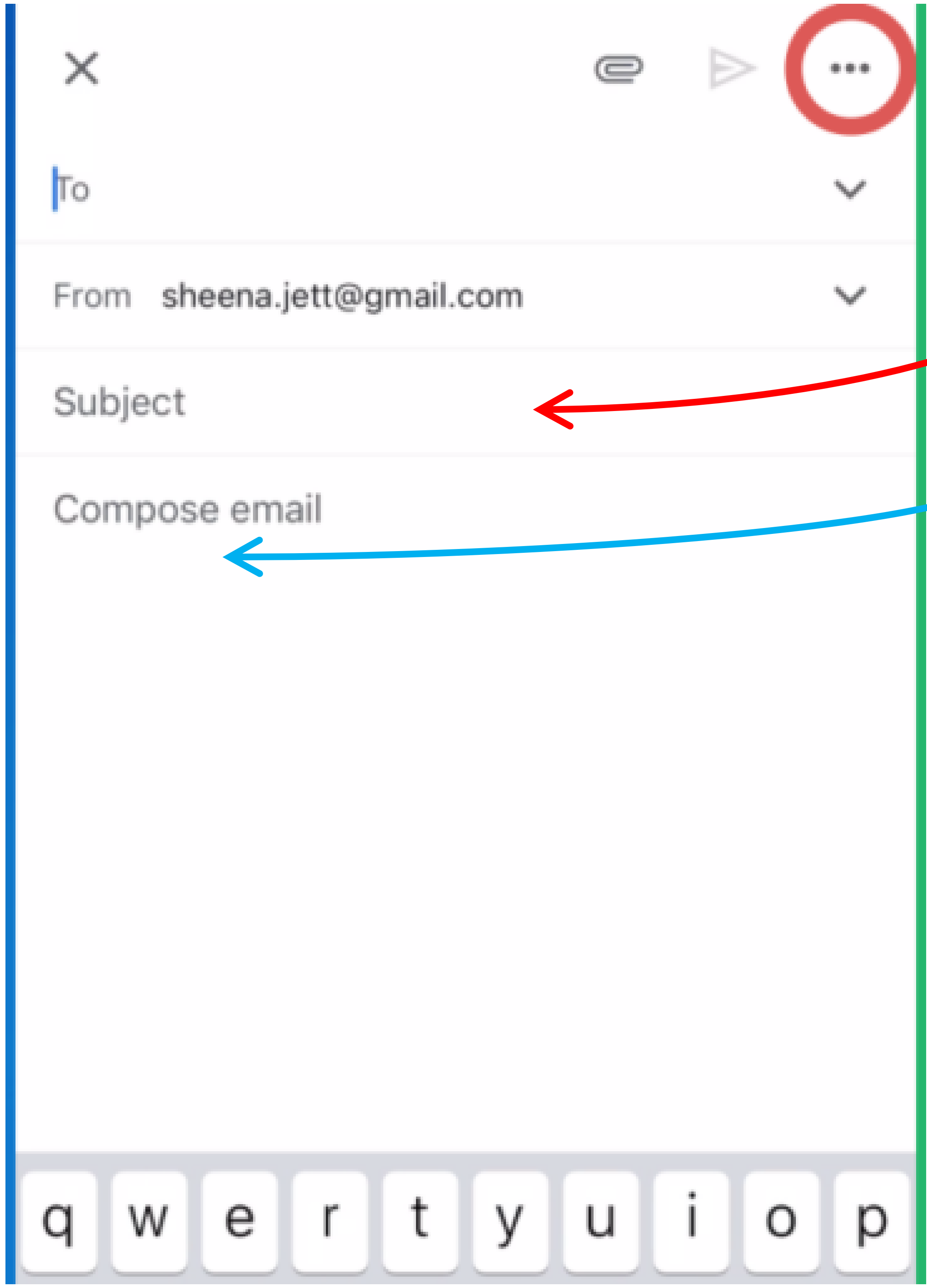
$$\sigma_{\text{comb},K} = \frac{P}{96\text{ mm}^2} - \frac{P}{4.73043\text{ mm}^2} = P\left[\frac{1}{96\text{ mm}^2} - \frac{1}{4.73043\text{ mm}^2}\right] = -\frac{P}{4.97560\text{ mm}^2}$$

$$\frac{P}{4.97560\text{ mm}^2} \leq 108\text{ MPa} = 108\text{ N/mm}^2 \qquad \therefore P \leq 537\text{ N}$$

Controlling Clamping Force The maximum allowable clamping force is **P = 537 N.** <u>(the smallest)</u>

طريقك إلى انجاز
YOUR WAY TO SUCCESS

HW1: Send an email to me ahmed.gheni@tu.edu.iq



الموضوع: معلومات الطالب الشخصيه

المحتوى:

السلام عليكم

د. احمد,ادناه ستجد اسمي الكامل ومرحلتي وشعبتي مع ارفاق الصوره الشخصية الرسميه.

احمد محمد عبد الله
المرحلة الثانية
شعبه B

مع التقدير